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ORTHOGONALITY OF GENERALIZED (σ,τ)SYMMETRIC BIDERIVATIONS IN SEMIPRIME RINGS

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ABSTRACT

The present article shows the relation between symmetric generalized (σ,τ) biderivations with orthogonality on semiprime rings and showed the orthogonality results. For (Δ_1, B_1) and (Δ_2, B_2) be two generalized (σ,τ) symmetric biderivations of R, then the following conditions are equivalent :

- ***Generalized (σ,τ)symmetric biderivations(Δ_1, B_1) and (Δ_2, B_2) are orthogonal.***
- ***For all $x, y, z \in R$, the following relations hold***
- $\Delta_1(x,y)\Delta_2(y,z)+\Delta_2(x,y)\Delta_1(y,z) = 0$.
- $B_1(x,y)\Delta_2(y,z) + B_2(x,y)\Delta_1(y,z) = 0$.
- $\Delta_1(x,y)\Delta_2(y,z) = B_1(x,y)\Delta_2(y,z) = 0$.
- $\Delta_1(x,y)\Delta_2(y,z) = 0$, ***for all*** $x, y, z \in R$***and***$B_1\Delta_2 = B_1B_2 = 0$.
- $(\Delta_1\Delta_2, B_1B_2)$***is a generalized*** (σ,τ) ***biderivation and*** $\Delta_1(x,y)\Delta_2(y,z) = 0$, ***for all*** $x, y, z \in R$.

KEYWORDS: Semiprime Ring, Symmetric Biderivation, Generalized symmetric Biderivation & Orthogonal Biderivation

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Original Article

INTRODUCTION

Vukman and Bresar [3] given the introduction of orthogonality for a couple of derivations d and g of a semiprime ring, and they obtained necessary and sufficient conditions for a couple of derivations are to be orthogonal and also they have given the associated results to a classical result of Posner [7]. Argac, Nakajima and Albas [1] have studied some results orthogonal generalized derivations of semiprime rings. Daifet. al. [4] given the orthogonality conditions for the derivations and biderivations of a ring and also in terms of a nonzero ideal of a two torsion free semiprime ring. Jaya Subba Reddy and Ramoorthy Reddy [5] obtained related results on symmetric biderivation swith orthogonality in Semiprime rings and obtained some essential conditions for two biderivations are to be orthogonal. Jaya Subba Reddy and Ramoorthy Reddy [6] obtainedsome results of generalized symmetric biderivations with orthogonality conditions of semiprime rings. In this current paper, we extended the results of generalized symmetric biderivations to (σ,τ) symmetric biderivations.

PRELIMINARIES

In the entire paper R represents associative ring. We call a ring R is two torsion free if $2a = 0, a \in R$implies a= 0. We call a ring R isprime if $aRb = 0$ implies a= 0 orb= 0 and is semiprime if $aRa = 0$ implies$a = 0$ .. We write $[x, y] = xy - yx$, for all $x, y \in R$ and the commutate or identities $[xy, z] = [x, z]y + x[y, z]$, $[x, yz] = [x, y]z + y[x, z]$. Let $d: R \to R$be additive map represents derivation if $d(xy) = d(x)y + xd(y)$, for all $x, y \in R$ .. Let $d: R \to R$be additive map represents Generalized derivation if $D(xy) = D(x)y + xd(y)$, for all $x, y \in R$. A mapping$B(.,.): R \times R \to R$represents symmetric if $B(x, y) = B(y, x)$, for all $x, y \in R$ .. A symmetric biadditive mapping $B(.,.): R \times R \to R$ represents a symmetric biderivation if $\mathrm{B}(xy, z) = B(x, z)y + xB(y, z), B(x, yz) = B(x, y)z + yB(x, z)$, for all $x, y, z \in R$. Let $B: R \times R \to R$ be a biderivation, a biadditive mapping $\Delta: R \times R \to R$ represents a generalized biderivationif $\Delta(xy, z) = \Delta(x, z)y + xB(y, z)$,, $\Delta(x, yz) = \Delta(x, y)z + yB(x, z)$, for all $x, y, z \in R$. We call an additive mapping $d: R \to R$ is called a (σ, τ) derivation if $d(xy) = \sigma(x)d(y) + d(x)\tau(y)$, for all $x, y \in R$. $B(.,.): R \times R \to R$is represents a symmetric(σ, τ) biderivation if $B(xy, z) = B(x, z)\sigma(y) + \tau(x)B(y, z)$, for all $x, y, z \in R$,where σ, τ are endomorphisms on R. similarly for generalized (σ, τ)symmetric biderivation if there is a (σ, τ) biderivation such that $\Delta(xy, z) = \Delta(x, z)\sigma(y) + \tau(x)B(y, z)$,, $\Delta(x, yz) = \Delta(x, y)\sigma(z) + \tau(y)B(x, z)$, for all $x, y, z \in R$. For a semiprime ring, a couple of derivations d and g are represents orthogonal if $d(x)Rg(y) = 0$ = g(y)Rd(x), for all $x, y \in R$ and two biderivations B and D represents orthogonal if $B(x, y)RD(y, z) = (0) = D(y, z)R\, B(x, y)$, for all $x, y, z \in R$. Two generalized biderivationsΔ_1 and Δ_2 associated with biderivations $B_1$and B_2 are called orthogonal if $\Delta_1(x, y)R\Delta_2(y, z) = (0) = \Delta_2(y, z)R\, \Delta_1(x, y)$, for all $x, y, z \in R$.

MAINTEXT

Theorem 1

For (Δ_1, B_1) and $(\Delta_2, \mathrm{B}_2)$are generalized(σ, τ)symmetric biderivations of R, then the following are equivalent to one another:

- Generalized(σ, τ)symmetric biderivations(Δ_1, B_1) and (Δ_2, B_2) are orthogonal.
- For all $\mathrm{x, y, z} \in \mathrm{R}$, the following relations hold.
  - $\Delta_1(\mathrm{x, y})\Delta_2(\mathrm{y, z})+\Delta_2(\mathrm{x, y})\Delta_1(\mathrm{y, z}) = 0$
  - $\mathrm{B}_1(\mathrm{x, y})\Delta_2(\mathrm{y, z}) + \mathrm{B}_2(\mathrm{x, y})\Delta_1(\mathrm{y, z}) = 0$.
- $\Delta_1(\mathrm{x, y})\Delta_2(\mathrm{y, z}) = \mathrm{B}_1(\mathrm{x, y})\Delta_2(\mathrm{y, z}) = 0$,, for all $\mathrm{x, y, z} \in \mathrm{R}$.
- $\Delta_1(\mathrm{x, y})\Delta_2(\mathrm{y, z}) = 0$,, for all $\mathrm{x, y, z} \in \mathrm{R}$and $\mathrm{B}_1\Delta_2 = \mathrm{B}_1\mathrm{B}_2 = 0$.
- $(\Delta_1\Delta_2, \mathrm{B}_1\mathrm{B}_2)$ is a generalized (σ, τ)symmetric biderivation and $\Delta_1(\mathrm{x, y})\Delta_2(\mathrm{y, z}) = 0$, for all $\mathrm{x, y, z} \in \mathrm{R}$.

For the proof of this theorem, we need some lemmas.

Lemma 1 ([3], lemma 1)

For a two torsion free semiprime ring $R$and a, b elements of R. Then the following are equivalent.

- $\mathrm{axb} = 0$

- $bxa = 0$
- $axb + bxa = 0$, for all $x \in R$.

  If one of above conditions is satisfied, then $ab = ba = 0$.

Lemma 2 ([5], Lemma2)

for a two torsion free semiprimering R. Suppose that two biadditive mappings $B: R \times R \to R$ and $D: R \times R \to R$ satisfy $B(x, y)RD(x, y) = (0)$, for all $x, y \in R$.

Then $B(x, y)RD(y, z) = (0)$, for all $x, y, z \in R$.

Lemma 3

If (Δ_1, B_1) and (Δ_2, B_2) are orthogonal generalized (σ, τ)symmetric biderivations of R, then the following relations hold.

- $\Delta_1(x, y)\Delta_2(y, z) = \Delta_2(x, y)\Delta_1(y, z) = 0$, hence $\Delta_1(x, y)\Delta_2(y, z) + \Delta_2(x, y)\Delta_1(y, z) = 0$, for all $x, y, z \in R$.
- B_1andΔ_2 are orthogonalorthogonal(σ, τ)symmetric biderivation and generalized (σ, τ)symmetric biderivation, and $B_1(x, y)\Delta_2(y, z) = \Delta_2(y, z)B_1(x, y) = 0$, for all $x, y, z \in R$.
- orthogonal orthogonal(σ, τ)symmetric biderivation and generalized (σ, τ)symmetric biderivation, and $B_2(x, y)\Delta_1(y, z) = \Delta_1(y, z)B_2(x, y) = 0$, for all $x, y, z \in R$.
- B_1andB_2 are orthogonal (σ, τ)symmetric biderivations.
- $B_1\Delta_2 = \Delta_2 B_1 = 0$ and $B_2\Delta_1 = \Delta_1 B_2 = 0$.
- $\Delta_1\Delta_2 = \Delta_2\Delta_1 = 0$.

Proof (i): Since (Δ_1, B_1) and (Δ_2, B_2) are orthogonal, we have $\Delta_1(x, y)r\Delta_2(y, z) = 0$, for all $x, y, z \in R$. By lemma 1, we get $\Delta_1(x, y)\Delta_2(y, z) = \Delta_2(y, z)\Delta_1(x, y) = 0$, then $\Delta_1(x, y)\Delta_2(y, z) + \Delta_2(y, z)\Delta_1(x, y) = 0$, hence condition (i) is proved.

Proof of (ii): Since (Δ_1, B_1) and (Δ_2, B_2) are orthogonal, we have $\Delta_1(x, y)r\Delta_2(y, z) = 0$, for all $x, y, z \in R$, implies $\Delta_1(x, y)\Delta_2(y, z) = 0$. Replacing x by rx in the above equation, we find that

$$\Delta_1(rx, y)\Delta_2(y, z) = 0$$

$$(\Delta_1(r, y)\sigma(x) + \tau(r)B_1(x, y))\Delta_2(y, z) = 0$$

$$\Delta_1(r, y)\sigma(x)\Delta_2(y, z) + \tau(r)B_1(x, y)\Delta_2(y, z) = 0.$$

Since by our hypothesis of orthogonality, we seen $\tau(r)B_1(x, y)\Delta_2(y, z) = 0$, on left multiply with $B_1(x, y)\Delta_2(y, z)$ to get $B_1(x, y)\Delta_2(y, z)\tau(r)B_1(x, y)\Delta_2(y, z) = 0$. Since $\tau(r) = r \in R$, by the semiprimeness of R, $B_1(x, y)\Delta_2(y, z) = 0$. Again replace x by xr in the above equation, it becomes,

$$B_1(xr, y)\Delta_2(y, z) = 0$$

$$(B_1(x, y)\sigma(r) + \tau(x)B_1(r, y))\Delta_2(y, z) = 0$$

$B_1(x,y)\sigma(r)\Delta_2(y,z) + \tau(x)B_1(r,y)\Delta_2(y,z) = 0$, which reduces to

$B_1(x,y)\sigma(r)\Delta_2(y,z) = 0$ and since $\sigma(r) = r \in R$, using Lemma 1, we get $B_1(x,y)\Delta_2(y,z) = 0$. Hence condition (ii) is proved.

Similarly we can prove condition (iii).

Proof of (iv): We have $\Delta_1(x,y)\Delta_2(y,z) = 0$, for all $x, y, z \in R$. Replace x by xr in expression, we find that $\Delta_1(xr,y)\Delta_2(y,z) = 0$

$$(\Delta_1(x,y)\sigma(r) + \tau(x)B_1(r,y))\Delta_2(y,z) = 0$$

$\Delta_1(x,y)\sigma(r)\Delta_2(y,z) + \tau(x)B_1(r,y)\Delta_2(y,z) = 0.$

Substituting z by zr in the above expression, we get

$\Delta_1(x,y)\sigma(r)\Delta_2(y,zr) + \tau(x)B_1(r,y)\Delta_2(y,zr) = 0.$

Since $\sigma(r) = r$ and $\tau(x) = x$, it reduces to $\Delta_1(x,y)r\Delta_2(y,zr) + xB_1(r,y)\Delta_2(y,zr) = 0$

$\Delta_1(x,y)r\Delta_2(y,z)\sigma(r) + \Delta_1(x,y)r\tau(z)B_2(y,r) + xB_1(r,y)\Delta_2(y,z)\sigma(r) + xB_1(r,y)\tau(z)B_2(y,r)=0.$

By using condition (ii) and (iii), we get

$xB_1(r,y)\tau(z)B_2(y,r)=0$, which implies $(B_1(r,y)\tau(z)B_2(y,r))x(B_1(r,y)\tau(z)B_2(y,r)) = 0.$

Since $\tau(z) = z$ and R is a semiprime, we get $B_1(r,y)zB_2(y,r) = 0$. Therefore we can write

$B_1(x,y)zB_2(x,y) = 0$. Using lemma 2, we have B_1 and B_2 are orthogonal biderivations.

Proof of (v): With the use of (ii) and (iv), we have $B_1(x,y)r\Delta_2(y,z) = 0$ and $\Delta_2(B_1(x,y)r\Delta_2(y,z),m) = 0$

$$\Delta_2(B_1(x,y),m)\sigma(r\Delta_2(y,z)) + \tau(B_1(x,y))B_2(r\Delta_2(y,z),m) = 0$$

$$\Delta_2 B_1(x,y)\sigma(r\Delta_2(y,z)) + B_1(x,y)rB_2(\Delta_2(y,z),m) + \tau(B_1(x,y))B_2(r,m)\Delta_2(y,z) = 0$$

Since B_1 and B_2 are orthogonal (σ,τ) biderivations and σ,τ are endomorphisms i.e $\sigma(r\Delta_2(y,z)) = r\Delta_2(y,z)$, $\tau(B_1(x,y)) = B_1(x,y)$, we have $\Delta_2 B_1(x,y)r\Delta_2(y,z) = 0$.

Replacing z by $B_1(x,y)z$ in the above equation, we have

$$\Delta_2 B_1(x,y)r\Delta_2(y, B_1(x,y)z) = 0$$

$\Delta_2 B_1(x,y)r\Delta_2(y, B_1(x,y))\sigma(z) + \Delta_2 B_1(x,y)r\tau(B_1(x,y))B_2(y,z) = 0.$

Since B_1 and B_2 are orthogonal (σ,τ) biderivations and σ,τ are endomorphisms

$\Delta_2 B_1(x,y)r\Delta_2(y, B_1(x,y))z = 0$, it changes to $\Delta_2 B_1(x,y)r\Delta_2(y, B_1(x,y))z\Delta_2 B_1(x,y)r\Delta_2(y, B_1(x,y)) = 0$. Using the semiprimeness of R, then $\Delta_2 B_1(x,y)r\Delta_2(y, B_1(x,y)) = 0$ which reduces to $\Delta_2 B_1(x,y)r\Delta_2 B_1(x,y) = 0$. Again use the semiprimeness, $\Delta_2 B_1(x,y) = 0$, in similar arguments we can prove $B_1\Delta_2 = \Delta_1 B_2 = B_2\Delta_1 = \Delta_1\Delta_2 = \Delta_2\Delta_1 = 0$.

Proof of Theorem 1

Already we seen (i) implies (ii), (iii), (iv) and (v) using lemma 3.

Next we prove (ii) implies (i)

$\Delta_1(x,y)\Delta_2(y,z)+\Delta_2(x,y)\Delta_1(y,z)=0$, substituting xr instead of x in the above expression, we find that $\Delta_1(xr,y)\Delta_2(y,z)+\Delta_2(xr,y)\Delta_1(y,z)=0$

$$\Delta_1(x,y)\sigma(r)\Delta_2(y,z)+\tau(x)B_1(r,y)\Delta_2(y,z)+\Delta_2(x,y)\sigma(r)\Delta_1(y,z)+\tau(x)B_2(r,y)\Delta_1(y,z)=0.$$

$$\Delta_1(x,y)\sigma(r)\Delta_2(y,z)+\Delta_2(x,y)\sigma(r)\Delta_1(y,z)+\tau(x)\big(B_1(r,y)\Delta_2(y,z)+B_2(r,y)\Delta_1(y,z)\big)=0$$

By our assumption (ii) of theorem, we have

$\Delta_1(x,y)\sigma(r)\Delta_2(y,z)+\Delta_2(x,y)\sigma(r)\Delta_1(y,z)=0$, when $r\in R$ and $\sigma(r)=r\in R$ which reduces to $\Delta_1(x,y)R\Delta_2(y,z)+\Delta_2(x,y)R\Delta_1(y,z)=0$. Thus by Lemma 1, we have Δ_1 and Δ_2 are orthogonal.

To prove (iii) implies (i)

$\Delta_1(x,y)\Delta_2(y,z)=B_1(x,y)\Delta_2(y,z)=0$, put xr in place of x in the above equation, we get

$$\Delta_1(xr,y)\Delta_2(y,z)=0$$

$\Delta_1(x,y)\sigma(r)\Delta_2(y,z)+\tau(x)B_1(r,y)\Delta_2(y,z)=0$, which reduces to

$$\Delta_1(x,y)\sigma(r)\Delta_2(y,z)=0$$

Therefore Δ_1 and Δ_2 are orthogonal generalized (σ,τ)symmetric biderivations, for some $\sigma(r)=r$.

To prove (iv) implies (i)

$B_1B_2=0$.

$$B_1\Delta_2(xy,z)=B_1\Big(\big(\Delta_2(x,z)\sigma(y)+\tau(x)B_2(y,z)\big),m\Big)$$

$$=B_1(\Delta_2(x,z)\sigma(y),m)+B_1(\tau(x)B_2(y,z),m)$$

$$=\tau(\Delta_2(x,z))B_1(\sigma(y),m)+B_1(\Delta_2(x,z),m)\sigma\sigma(y)+B_1(x,m)B_2(y,z)+\tau\tau(x)B_1(B_2(y,z),m)$$

Since σ,τ are endomorphisms, then the expression gives

$B_1\Delta_2(xy,z)=\Delta_2(x,z)B_1(y,m)+B_1(\Delta_2(x,z),m)y+B_1(x,m)B_2(y,z)+xB_1(B_2(y,z),m)$.

With our assumption $\Delta_2(x,z)B_1(y,m)=0$, then replace x by xr, we find that $\Delta_2(xr,z)B_1(y,m)=0$

$$\Delta_2(x,z)\sigma(r)B_1(y,m)+\tau(x)B_2(r,z)B_1(y,m)=0$$

$$\Delta_2(x,z)\sigma(r)B_1(y,m)=0$$

By Lemma 1 and Lemma 3, using $\sigma(r)=r$, we obtain $\Delta_2(x,z)B_1(y,m)=0$or $B_1(x,y)\Delta_2(y,z)=0$, then Δ_1 and Δ_2 are orthogonal generalized (σ,τ)symmetric biderivations.

To prove (v) $\Longrightarrow$ (i)

Since $(\Delta_1\Delta_2,B_1B_2)$ is a generalized (σ,τ) symmetric biderivation with B_1B_2 is a (σ,τ)biderivation. Then we get

$\Delta_1\Delta_2(xy,z)=\Delta_1\Delta_2(x,z)y+xB_1B_2(y,z)$, for all $x,y,z\in R$.

We have $\Delta_1\Delta_2(xy,z)=\Delta_1(\Delta_2(xy,z),m)$

$$=\Delta_1(\Delta_2(x,z)\sigma(y)+\tau(x)B_2(y,z),m)$$

$$=\Delta_1(\Delta_2(x,z)\sigma(y),m)+\Delta_1(\tau(x)B_2(y,z),m)$$

$$=\Delta_1(\Delta_2(x,z),m)\sigma\sigma(y)+\tau(\Delta_2(x,z))B_1(\sigma(y),m)+$$

$$\Delta_1(\tau(x),m)\sigma(B_2(y,z))+\tau\tau(x)B_1(B_2(y,z),m)$$

Since σ,τ are endomorphisms, then the expression gives $\Delta_2(x,z)B_1(y,m)+\Delta_1(x,m)B_2(y,z)=0$. Then replace $m$by z, we get $\Delta_2(x,z)B_1(y,z)+\Delta_1(x,z)B_2(y,z)=0$, this can be written as

$$\Delta_2(x,y)B_1(y,z)+\Delta_1(x,y)B_2(y,z)=0. \qquad (1)$$

Since $\Delta_1(x,y)\Delta_2(y,z)=0$

Replace z by mz in the above expression, we get

$$\Delta_1(x,y)\Delta_2(y,mz)=0$$

$$\Delta_1(x,y)\Delta_2(y,m)\sigma(z)+\Delta_1(x,y)\tau(m)B_2(y,z)=0$$

$\Delta_1(x,y)\tau(m)B_2(y,z)=0$, therefore $\Delta_1(x,y)B_2(y,z)=0$. Then Eq.(1) becomes $\Delta_2(x,y)B_1(y,z)=0$, for all $x,y,z\in R$.

Replace z by zm in the above expression gives that, $\Delta_2(x,y)B_1(y,zm)=0$

$$\Delta_2(x,y)B_1(y,z)\sigma(m)+\Delta_2(x,y)\tau(z)B_1(y,m)=0$$

$\Delta_2(x,y)\tau(z)B_1(y,m)=0$. Therefore B_1 and Δ_2 are orthogonal. Since from lemma 3, we have Δ_1 and Δ_2 are orthogonal.

GENERALIZED(σ,τ)SYMMETRIC BIDERIVATIONS-ITS PRODUCTS

Theorem 2

For generalized (σ,τ)symmetric biderivations(Δ_1,B_1),(Δ_2,B_2) of R. Then the following conditions are equivalent

- $(\Delta_1\Delta_2,B_1B_2)$ is a generalized (σ,τ) symmetric biderivation.
- $(\Delta_2\Delta_1,B_2B_1)$ is a generalizeds (σ,τ) symmetric biderivation.
- $\Delta_1$and B_2 are orthogonal, and Δ_2 and B_1 are orthogonal.

Proof: To prove (i) $\Longrightarrow$ (iii):

Assume that $(\Delta_1\Delta_2,B_1B_2)$ is a generalized (σ,τ)symmetric biderivation. Thus, as in the proof of theorem 1 (v) $\Longrightarrow(ii)$, we have

$\Delta_1(x,y)B_2(y,z)+\Delta_2(x,y)B_1(y,z)=0$, for all $x,y,z\in R$.

Replace z by zm in the above expression, we find that $\Delta_1(x,y)B_2(y,zm)+\Delta_2(x,y)B_1(y,zm)=0$

$$\Delta_1(x,y)B_2(y,z)\sigma(m) + \Delta_1(x,y)\tau(z)B_2(y,m) + \Delta_2(x,y)B_1(y,z)\sigma(m) + \Delta_2(x,y)\tau(z)B_1(y,m)$$

$$= 0\{\Delta_1(x,y)B_2(y,z) + \Delta_2(x,y)B_1(y,z)\}\sigma(m) + \Delta_1(x,y)\tau(z)B_2(y,m) + \Delta_2(x,y)\tau(z)B_1(y,m) = 0$$

$\Delta_1(x,y)\tau(z)B_2(y,m) + \Delta_2(x,y)\tau(z)B_1(y,m) = 0$, for all $x, y \in R$.

Again replace z by $B_1(x,y)z$ in the above expression, we find that

$$\Delta_1(x,y)\tau(B_1(x,y)z)B_2(y,m) + \Delta_2(x,y)\tau(B_1(x,y)z)B_1(y,m) = 0$$

Since σ,τ are endomorphisms and $(\Delta_1\Delta_2, B_1B_2)$ is a generalized (σ,τ)symmetric biderivation, B_1B_2 is (σ,τ)biderivation. Therefore B_1 and B_2 are orthogonal.

$\Delta_2(x,y)B_1(x,y)zB_1(y,m) = 0 = \Delta_2(x,y)B_1(x,y)z\Delta_2(x,y)B_1(y,m)$

$\Delta_2(x,y)B_1(x,y)R\Delta_2(x,y)B_1(y,m) = 0$. Since R is semiprimeness, we have

$\Delta_2(x,y)B_1(y,x) = 0$, then $\Delta_2(x,y)mB_1(y,z) = 0$,, for all $x, y, z, m \in R$. So

$\Delta_2$andB_1 are orthogonal, Similarly Δ_1 and B_2 are orthogonal.

To prove (iii) ⟹ (i): Since Δ_1 and B_2 are orthogonal, we have

$\Delta_1(x,y)rB_2(y,z) = 0$, $\Delta_1(x,y)B_2(y,z) = 0$ and $\Delta_2(x,y)B_1(y,z) = 0$.

Therefore $\Delta_1\Delta_2(xy,z) = \Delta_1\Delta_2(x,z)y + xB_1B_2(y,z)$.

Thus $(\Delta_1\Delta_2, B_1B_2)$is a generalized symmetric biderivation.

(ii) ⟹ (iii) and (iii) ⟹ (ii) is proved in a similar way.

CONCLUSIONS

- From Theorem 1, we conclude a couple, generalized (σ,τ)symmetric biderivations of R satisfies equivalent conditions from (i) to (v).
- From Theorem 2, we can find $\Delta_1\Delta_2$, $\Delta_2\Delta_1$, orthogonality of Δ_2,B_1 and orthogonality of $\Delta_1 B_2$ are equivalent.

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